EFFECTIVENESS OF GAS FILM COOLING FOR A TURBULENT BOUNDARY LAYER

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An effective hydrodynamic method for protecting mechanical parts (combustion chamber walls, engine nozzles, etc.) from the effect of high-temperature gas flows is the gas film. A gas film can be formed, for example, by injecting a cooling gas through a porous zone or through one or several slots in the initial section of the body to be cooled. The effectiveness of the gas film has been the subject of a number of theoretical and experimental studies [1–11]. Below, we offer a solution which makes it possible to calculate a number of cases by means of a single method. The analysis is based on the physical model proposed in [4]: when a coolant is blown through a porous zone (Fig. 1a) or a slot (Fig. 1b), in the section $\Delta x = 0$ (starting from which the effectiveness θ decreases) there exists a developed boundary layer with corresponding values of the energy thickness δ_{e0}^{**} and momentum thickness δ_0^{**}. The solution obtained provides a satisfactory generalization of the experimental data reported by various authors. An analysis of the theoretical formulas and experimental data shows that the method used for injecting the coolant does not have an appreciable effect on effectiveness.

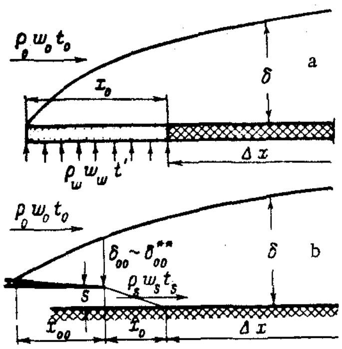

Fig. 1. Methods of creating a gas film.

The effectiveness is determined by the adiabatic wall temperature. The corresponding formula is derived below.

1. Let a plate be situated in a flow of hot gas. The initial section of the plate is cooled by a cold gas in such a manner that the wall temperature at $x < x_0$ is constant. The adiabatic portion of the plate $x > x_0$ (Fig. 1) is protected from the hot gas stream by the relatively cool gas in the boundary layer.

The heat flux that must be supplied to the region $x > x_0$ in order to achieve a wall temperature equal to that of the main flow is [5]

$$q_{w1} = \alpha_x (t_{w0} - t_0) + f(x, x_0)\, \alpha_{\Delta x} (t_0 - t_{w0}), \qquad (1.1)$$

where α_x and $\alpha_{\Delta x}$ are the heat transfer coefficients at the plate for t^{ω} = const.

This same heat flux can also be written in terms of the temperature of a thermally insulated wall as

$$q_{w1} = \alpha_1 (t_0 - t_{aw}). \qquad (1.2)$$

The heat transfer coefficients are determined from the heat transfer law [3]

$$S = \frac{A}{R^{**-m_1}} P^{-0.75}, \qquad (1.3)$$

where S is the Stanton number, R^{**} is the Reynolds number with respect to energy thickness, P is the Prandtl number; thus, for a power-law velocity profile with exponent $n = 1/7$, we have [3] $A = 0.0128$ and $m_1 = 0.25$.

The equations (1.1) through (1.3) yield

$$\frac{\theta}{R_1^{**m_1}} = \frac{1}{R_{\Delta x}^{**m_1}} f(x, x_0) - \frac{1}{R_x^{**m_1}} \qquad \left(\theta = \frac{t_0 - t_{aw}}{t_0 - t_{w0}}\right). \qquad (1.4)$$

The function $f(x, x_0)$ has the form [3]

$$f(x, x_0) = \left[\frac{A(m_1+1) R_{\Delta x}}{R_0^{**(m_1+1)} + A(m_1+1) R_{\Delta x}}\right]^{0.086}. \qquad (1.5)$$

Values of R_x^{**} and $R_{\Delta x}^{**}$ are found from the solution of the energy equation for the plate at t_ω = const, and values of $R_{\Delta x}^{**}$ and R_x^{**} from the solution of the momentum equation. When $P \approx 1.0$ we have

$$R_x^{**} = [R_0^{**(m_1+1)} + A(m_1+1) R_{\Delta x}]^{1/m_1+1},$$
$$R_{\Delta x}^{**} = [A(m_1+1) R_{\Delta x}]^{1/(m_1+1)}, \qquad (1.6)$$

$$R_x^{**} = [R_0^{**(m_1+1)} + A(m_1+1) R_{\Delta x}]^{1/m_1+1},$$
$$R_{\Delta x}^{**} = [A(m_1+1) R_{\Delta x}]^{1/(m_1+1)}. \qquad (1.7)$$

When $x \to \infty$, we may write

$$\int_{x_0}^{x} q_{w1}\, dx = \rho_0 w_0 c_{p0} (t_0 - t_{w0})\, \delta_{e0}^{**}. \qquad (1.8)$$

Hence,

$$R_1^{**} = \frac{R_0^{**}}{\theta}. \qquad (1.9)$$

With the aid of (1.5), (1.7), and (1.9), we obtain from Eq. (1.4) a formula for the dimensionless adiabatic wall temperature (effectiveness)

$$\theta = \left[\left(\frac{R_x^{**}}{R_{\Delta x}^{**}}\right)^{m_1} f(x, x_0) - 1\right]^{1/m_1+1} \left(\frac{R_0^{**}}{R_x^{**}}\right)^{m_1/m_1+1}. \qquad (1.10)$$

2. If the cooling gas is injected through a porous zone at the front of the plate (Fig. 1a), then, for supercritical injection, the energy equation for the region $x < x_0$ will have the form [3]

$$\frac{dR_e^{**}}{dR_x} - \frac{c_p' \rho_w w_w}{c_{p0} \rho_0 w_0} = 0. \qquad (2.1)$$

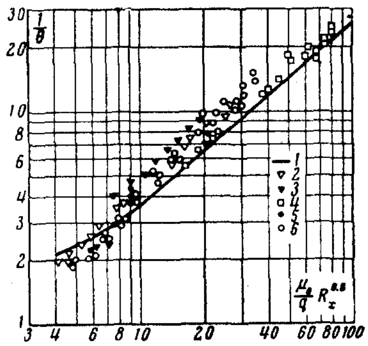

Fig. 2. Coolant injection through a porous zone: 1 refers to calculations from formula (2.5); the points 2 through 6 refer to experiments in [8].

Hence, at $c_p' \rho_w w_w / c_{p0} \rho_0 w_0 = \text{const}$, at the point $x < x_0$ we obtain the following value

$$R_0^{**} = \frac{\rho_w w_w}{\mu_0} x_0 = \frac{q}{\mu_0}, \tag{2.2}$$

where q is the amount of injected gas per unit width of surface. If injection is subcritical, the value of R_0^{**} is found from the design of the porous zone [3]. In this case

$$R_0^{**} = \frac{q}{\mu_0} \frac{t_0 - t'}{t_0 - t_{w0}}. \tag{2.3}$$

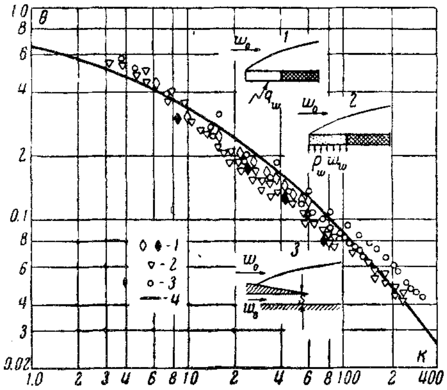

Fig. 3. Generalization of experimental data on effectiveness for various injection methods: 1 refers to experiments in [7], 2 to experiments in [8], 3 to experiments at m ≤ 0.2 in [10], 4 to calculations based on (3.4).

Since, in the case of heat transfer at a porous wall, the momentum thickness equals the energy thickness [3], in the section $x = x_0$

$$R_0^{**} = R_0^{**}. \tag{2.4}$$

Then Eq. (1.10) for supercritical injection through a porous zone will take the form

$$\theta = \frac{t_0 - t_{aw}}{t_0 - t'} = \frac{\{[1 + 62.5(\mu_0 q^{-1} R_{\Delta x}^{0.8})^{-1.25}]^{0.114} - 1\}^{0.8}}{[1 + 0.016(\mu_0 q^{-1} R_{\Delta x}^{0.8})^{1.25}]^{0.16}}. \tag{2.5}$$

It can be seen from Fig. 2, in which calculations based on (2.5) are compared with the experimental data in [8], that the theoretical formula gives a satisfactory description of the experimental data.

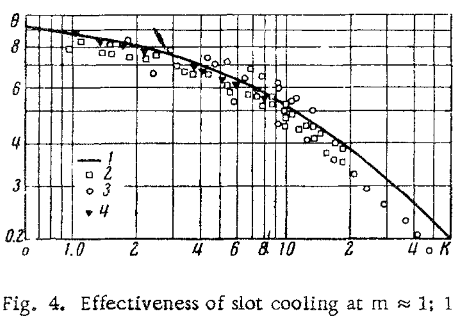

Fig. 4. Effectiveness of slot cooling at m ≈ 1; 1 refers to calculations based on (3.5), 2 to experiments in [9], 3 to experiments in [10], and 4 to experiments in [11].

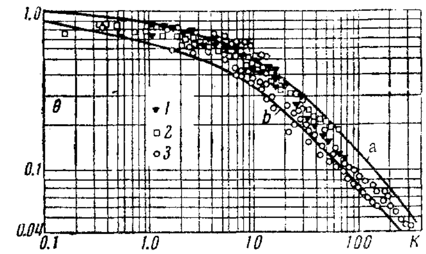

Fig. 5. Coolant injection through a tangential slot 0 < m ≤ 1; curves a and b refer to calculations based on (3.5) and (3.4), respectively, points 1 refer to experiments in [6, 11], points 2 to experiments in [9], points 3 to experiments in [10].

3. Let the cooling gas be injected through a single slot (Fig. 1b). In this case, in the section of the slot

$$\delta_{e0}^{**} = \int_0^\infty \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{t - t_w}{t_0 - t_w}\right) dy = \frac{\rho_s w_s}{\rho_0 w_0} s,$$

$$\delta_0^{**} = \int_0^\infty \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{w}{w_0}\right) dy = \frac{\rho_s w_s}{\rho_0 w_0}\left(1 - \frac{w_s}{w_0}\right) s. \tag{3.1}$$

Taking (3.1) into account, Eq. (1.10) for the "effectiveness" θ takes the form ($p_s \approx p_0$)

$$\theta = \frac{\{[1 + 62.5K^{-1}]^{0.2}[1 + 62.5K^{-1}|1 - m|^{1.25}]^{-0.086} - 1\}^{0.8}}{[1 + 0.016K]^{0.16}}$$

$$\left(\theta = \frac{t_0 - t_{0w}}{t_0 - t_s}, \quad K = \frac{\Delta x}{ms} R_s^{-0.25} = \frac{R_{\Delta x}}{R_{e0}^{**1.25}}, \quad m = \frac{\rho_s w_s}{\rho_0 w_0}\right). \tag{3.2}$$

Most of the experiments involving slot cooling were conducted in the presence of a developed boundary layer in the main flow. The method proposed makes it possible to take into account this effect in deriving Eq. (1.10); in this case one gets

$$\frac{\theta'}{\theta} = \left[\frac{1 + 0.016K}{(1 + \delta_{00}^{**} m^{-1} s^{-1})^{1.25} + 0.016K}\right]^{0.07}. \tag{3.3}$$

where θ and θ' are the values of the effectiveness with and without an initial boundary layer, δ_{00}^{**} is the momentum thickness above the slot. Formula (3.3) corresponds well with Seban's experimental data.

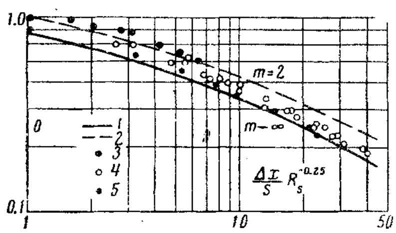

Fig. 6. Effectiveness at m > 1; points 1 and 2 refer to calculations based on (3.6) and (3.2), respectively, 3 to experiments at m ≈ 2 in [10], 4 to experiments at 2 < m < 10 in [10], 5 to experiments at m ≈ 21 in [10].

In (3.2) it is not possible to isolate a complex that would permit generalization of the experimental data. Accordingly, we shall examine three limiting cases, for which it is possible to obtain interpolation formulas.

(a) For m $\ll 1$, from (3.2) we obtain the following expression for θ:

$$\theta = \{[1 + 62\,(K + 0.143)^{-1}]^{0.114} - 1\}^{0.8}\;[1 + 0.016K]^{-0.16} \cdot \qquad (3.4)$$

A comparison of calculations based on (3.4) and experimental data in [7, 8, 10] is given in Figs. 3 and 5. It can be seen that (3.4) is in satisfactory agreement with experiment. Data from [7, 8] are plotted in Fig. 3. The experimental values of the effectiveness are well generalized by the same parameter regardless of the method used to create the gas film.

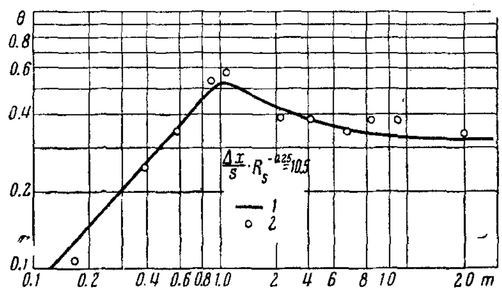

Fig. 7. Effectiveness of barrier cooling as a function of the blowing parameter: 1 refers to calculations based on (3.2), 2 refers to experiments in [10].

(b) For m ≈ 1, Eq. (3.2) takes the form

$$\theta = \{[1 + 62.5\,(K + 2)^{-1}]^{0.2} - 1\}^{0.8}\,(1 + 0.016K)^{-0.16}. \qquad (3.5)$$

From Figs. 4 and 5 it can be seen that this formula gives a satisfactory description of the experimental data in [6, 9–11].

(c) For m $\to \infty$, it is possible to derive an interpolation formula:

$$\theta = \left\{\left[1 + \frac{62.5}{s^{-1}\,\Delta x R_s^{-0.25} + 0.143}\right]^{0.114} - 1\right\}^{0.8}. \qquad (3.6)$$

Figure 6 gives a comparison between the experimental data in [10], calculations based on (3.6), and calculations based on (3.2) for m = 2.

Based on an analysis of experimental data, Seban [10] showed that with increasing blowing parameter m, the effectiveness rises to a maximum at m ≈ 1, while with subsequent decrease in m, it falls, asymptotically approaching the value at m ≈ 0.6. An analytical dependence of the effectiveness θ on the blowing parameter can be derived from formula (3.2). A comparison of calculations based on (3.2) with Seban's experimental data is given in Fig. 7. The calculations predict the same behavior of θ with variation of m as was observed in the experiment.

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